FIRST ERA

YEAR	NAME	CONTRIBUTION	
		FUNDAMENTAL PRINCIPLES	
1848	Sir William Thomson	Expressions for the displacements elicited by concentrated static forces acting at some arbitrary point in an elastic infinite solid.	
1849	George Gabriel Stokes	Extends previous solution to time varying point forces in an infinite medium, known as Stokes solution.	
1878	Joseph Valentin Boussinesq	A solution method for static vertical point loads applied onto the surface of an elastic half-space, and also gave a closed-form solution for a rigid disk with smooth contact on the circular surface of a half-space bearing uniformly distributed vertical loads.	$K_z = \frac{4Ga}{1-\nu}$ $\sigma_z = \frac{P_z}{2\pi a\sqrt{s^2-r^2}}$
1882	Valentino Cerruti	Obtains the response in the interior of an arbitrary solid due to forces prescribed on parts of the external boundaries (known today as a boundary value problem) by tractions (Neumann boundary) or displacements (Dirichlet boundary). Applies his method to a body of infinite extent limited by a flat surface (i.e. a half-space). His equations contain tools necessary to obtain a solution for the displacement field due to a distributed tangential load.	
1904	Horace Lamb	Solution for a homogeneous half-space subjected to a dynamic load on its surface, known as Lamb's problem. Precursors to what constitutes the modern integral transform method to obtain the response to either impulsive (2-D) or suddenly applied (3-D) vertical loads on the surface of an elastic half-space.	
1939	Cagnaird	Evaluated double integral transforms in Lamb's problems	
1960	De Hoop	Simplification of Cagnaird's method, known as *Cagnaird-de Hoop method*.	
1955 / 1960	Pekeris / Chao	Used to obtain closed-form solutions for impulsive vertical and horizontal point loads in a half-space for Poisson ratio of 0.25	
1974	Mooney	Generalized solution for vertical point loads in a half-space with arbitrary Poisson's ratio for the horizontal component	
1936	Raymond David Mindlin	Set of closed-form equations for the displacement field elicited by static, vertical and horizontal point loads buried at an arbitrary depth below the surface of an elastic half-space	
		STATIC SSI	
1893	Fr. Engesser	Wrote Zur Theorie des Baugrundes (The theory of soils), which discusses the stability and carrying capacity of foundations	
1926	Ferdinand Alois Schleicher[1]	Determines a coefficient, inversely proportional to linear dimensions of supported loads, which is useful in formulation of foundation mechanics problems via distributed Winkler springs. Determines closed-form formulas for loads distributed over a rectangular area; observes that the smallest deflection is observed at the four corners and equals one half of the deflection at the center for any aspect and Poisson ratios.	
1934	Wilhelm Steinbrenner[1]	Determined vertical stresses anywhere in the soil can readily be inferred from the stresses underneath the symmetry center of the loaded area, and from there to the corners (obtained from simple integration of the Boussinesq solution for point loads). Stresses elsewhere then follow by simple superposition of appropriately siz ed rectangular loads, including negative loads in the case of observation points beyond the edges of the actual load.	
1943	H. Borowicka[1]	Obtained stress distribution under strip footings and circular disks subjected to eccentric vertical loads; widely used formulas for rocking stiffness (K_r).	$K_{r-disk} = \frac{4Ga^3}{3(1-\nu)}$, $\sigma_{r-disk} = \frac{3r\cdot\cos\theta\cdot Mr}{2\pi a^3\sqrt{a^2-r^2}}$ $K_{r-strip} = \frac{4Ga^2}{2(1-\nu)}$, $\sigma_{r-strip} = \frac{2x\cdot Mr}{\pi a^2\sqrt{a^2-x^2}}$
1944	Reissner[1] and Sagoci	Torsional stiffness (K_t) of a circular plate welded to an elastic half-space.	$K_t = \frac{16Ga^3}{3}$, $\sigma_z = \frac{3r\cdot Mt}{4\pi a^3\sqrt{a^2-r^2}}$
1949	Raymond David Mindlin	derived lateral stiffness (K_h) of a rigid circular disk subjected to tangential/horizontal loads (i.e. swaying).	$K_h = \frac{8Ga}{2-\nu}$, $\tau_{xy} = \frac{P_x}{2\pi a\sqrt{a^2-r^2}}$
		DYNAMIC SSI	
1931	Karl Maguerre	Dealt with harmonically loaded soils, but was overwhelmed by the complexities of wave propagation and his developments did not base further works.	
1936	Erich Reissner[2]	Explores the behavior of circular disks on elastic half-spaces subjected to time-harmonic vertical loads. Assumed the plate to have frictionless contact with the soil and uniform stress distribution below the plate.	
1937	Erich Reissner[2]	Addresses the problem of an elastic half-space excited at the surface by concentrated and distributed torsional sources. Considered point moments, ring moments, distributed torsional disk sources. Evaluated the torsional response of massive cylinders. Considered finite depth layers and half-space soil configurations. Notable insights about radiation damping and equivalent mass-spring-damper analogy.	
1944	Erich Reissner[2] and Sagocy	Very first rigorous solution to a mixed boundary value problem involving a dynamically loaded plate. Used oblate spheroidal coordinates to find exact expressions for rigid circular plates loaded in torsion at arbitrarily high frequencies, however their solution lacked simple functions. Exact formulas for rigid spheres.	
1956	Bycroft	Assessed the cases of dynamically loaded circular plates resting on half-spaces and on strata of finite depth. Considers all four modes of vibration and assumes that the stress distribution can be approximated by the	

[1] A significant number of early/leading papers on soil mechanics and SSI emanated from the Austrian-German group at the Institute of Soil Mechanics, Technical University of Vienna, prior to WW II. Karl Terzaghi, its first director during the 1930's, is now regarded as the father of soil mechanics.

[2] Besides his pioneering work on SSI, Reissner was also very well known for his contribution to the theory of the Reissner Plate and to the Hellinger–Reissner Variational Principle.

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		static case. Restricts the analysis to fairly low frequencies.
1963	Thomson and Kobori	Rectangular foundations subjected to vertical loads, resting on half-spaces and on strata of finite depth. Limited the study to low frequencies, assuming smooth plate and uniform stresses.
1965	Awojobi and Grootenhuis	2-D strip footings subjected to vertical loads, resting on half-spaces and on strata of finite depth.
1967	Chadwick and Trowbridge	Solved the problem of a rigid sphere in a full-space subjected to torsion, in frequency and time domains. Additional solution for lateral or vertical loads, also in both frequency and time domains .
1976	Apsel and Luco	Exact solution to the torsional response of both prolate and oblate ellipsoidal foundations embedded in an elastic half-space, subjected to harmonic torque about the vertical axis and to SH waves propagated along arbitrary directions.
INTERACTION EFFECT WITHIN AND NEAR THE STRUCTURE		
1935	Sezawa and Kanai	Modeled a thin cylindrical rod with a hemispheric tip at the bottom which is embedded in a homogenous half-space, subjected to vertically propagated P-waves. Found that resonance effect remains limited due to loss of energy in the soil (SSI beneficial) and plotted amplification functions.
1940	Romeo Martel	Evaluated Hollywood Storage Building during 1933 Long Beach earthquake. Concluded that damage to building in soft soils, deep alluvia or high elevations are more widespread than those observed on building supported on firm ground. Could not confirm his predictions.
1954	Merrit and Housner	Analyzing horizontal records in basements and parking lots concluded that lateral compliance of the foundation has little effect. Beneficial effects of rocking depend on the ground motion and the height of the building, noticed by evaluating a rigid block on rotational spring (assessed with an analog computer).
1957	Housner	Evaluating the Hollywood Storage Building noticed that waves in the ground propagating along the long direction suffered significant filtering, which did not happen with waves in the perpendicular direction, demonstrating the effect lately known as kinematic interaction.
1967	Parmelee	Evaluated a simple 3 DOF structural model, mounted on lateral and rocking springs defined based on Bycroft's stiffness functions. Considered frequency dependence of stiffness functions.
1969	Nathan Newmark	Noticed a torsional effect in symmetric buildings excited by waves passing underneath the foundation, due to the difference in the time of arrival (therefore excitation) between different parts of the foundation. Called this phenomenon the Tau Effect.
1970	Sarrazín	Adopted Parmelee model evaluated the effect of the height of the center of mass of the foundation above the line of actions of the soil springs. Used frequency dependant spring-dampers systems. Found typically low values for rocking damping and high values for swaying damping. SSI is beneficial reducing response amplitudes, and hysteretic damping is important.
1976	Robert Scanlan	Continued in more depth the Newmark's work about the Tau Effect, producing a well know large study on the subject.
1977	Anestis Veletsos	Interacting systems can be modeled via simple systems with modified periods and appropriate levels of damping, rocking is important, similar translation at base and free field, SSI increases damping and reduces amplitude in the response, hysteretic damping is essential, effective damping is less in taller structures.