

STATIC APPROACH

DYNAMIC APPROACH

**1840** **Sir William Thomson**  
Expressions for the displacements elicited by concentrated static forces acting at some arbitrary point in an elastic infinite solid

**1849** **Sir George Gabriel Stokes**  
Extends Thomson's solution to time varying point forces in an infinite medium, known as **Stokes solution**

**1878** **Joseph V. Boussinesq**  
Solutions for static vertical point loads and rigid disk applied onto an elastic half-space  
$$K_z = \frac{4Ga}{1-\nu} \quad \sigma_z = \frac{P_z}{2\pi a\sqrt{s^2-r^2}}$$

**1882** **Valentino Cerruti**  
Response in the interior of an arbitrary solid due to forces prescribed on parts of the external by tractions or displacements

**1893** **Fr. Engesser**  
Stability and carrying capacity of foundations ("The theory of soils")

**1926** **Ferdinand Alois Schleicher**  
Confirms Boussinesq results. Infers the Modulus of Subgrade Reaction. Formulas for loads distributed over rectangular areas. For displacements:  $\delta_{corner} = \frac{1}{2} \delta_{center}$

**1934** **Wilhelm Steinbrenner**  
Vertical stresses anywhere from stresses underneath the symmetry center of the loaded rectangular area, and from there to the corners.

**1943** **H. Borowicka**  
Stress distribution under strip footings and circular disks subjected to eccentric vertical loads; formulas for rocking stiffness (Kr)

Disk	$K_r = \frac{4Ga^3}{3(1-\nu)}$	$\sigma_r = \frac{3r \cdot \cos \theta \cdot Mr}{2\pi a^3\sqrt{a^2-r^2}}$
Strip	$K_r = \frac{4Ga^2}{2(1-\nu)}$	$\sigma_r = \frac{2x \cdot Mr}{\pi a^2\sqrt{a^2-x^2}}$

**1944** **Reissner and Sagoci**  
Torsional stiffness (Kt) of a circular plate welded to an elastic half-space  
$$K_t = \frac{16Ga^3}{3} \quad \sigma_z = \frac{3r \cdot Mt}{4\pi a^3\sqrt{a^2-r^2}}$$

**1949** **Raymond David Mindlin**  
Lateral stiffness (Kh) of a rigid circular disk subjected to tangential/horizontal loads  
$$K_h = \frac{8Ga}{2-\nu} \quad \tau_{xy} = \frac{P_x}{2\pi a\sqrt{a^2-r^2}}$$

**1974** **Mooney**  
Generalized solution for vertical point loads in a half-space with arbitrary Poisson's ratio for the horizontal component.

**1904** **Horace Lamb**  
Solution for a homogeneous half-space subjected to a dynamic load on its surface, known as **Lamb's problem**.

**1931** **Karl Maguerre**  
Attempt to deal with harmonically loaded soils, limited by complexity of the problem

**1936** **Cagnaird / Erich Reissner**  
Double integral transforms in Lamb's problems / Analyzed circular disks on elastic half-spaces subjected to time-harmonic vertical loads.

**1937** **Erich Reissner**  
Elastic half-space excited at the surface by concentrated and distributed torsional sources. **Radiation damping and equivalent mass-spring-damper analogy**

**1944** **Reissner and Sagocy**  
Rigorous solution to mixed boundary value problem with dynamically loaded plate.

**1956** **Bycroft**  
Dynamically loaded circular plates resting on half-spaces and on strata of finite depth.

**1960** **De Hoop / Pekeris-Chao**  
Cagnaird-de Hoop simplified method / Impulsive vertical and horizontal point loads in a half-space for Poisson ratio of 0.25

**1963** **Thomson and Kobori**  
Rectangular foundations subjected to vertical loads, resting on half-spaces and on strata.

**1965** **Awojobi and Grootenhuis**  
2-D strip footings subjected to vertical loads

**1967** **Chadwick and Trowbridge**  
Rigid sphere in a full-space subjected to torsion, in frequency and time domains.

**1976** **Apsel and Luco**  
Exact solution to the torsional response of ellipsoidal in an elastic half-space.

**1935** **Sezawa and Kanai**  
Resonance effect remains limited due to loss of energy in the soil (SSI beneficial). Plotted amplification functions.

**1940** **Romeo Martel**  
Damage to building in soft soils, deep alluvia or high elevations are higher than in those supported on firm ground

**1954** **Merrit and Housner**  
Lateral compliance has little effect. Effects of rocking depend on the ground motion and the height of the building

**1957** **Housner**  
Waves in the ground propagating along the long direction of the building had more filtering. Known as **Kinematic Effect**

**1967** **Parmelee**  
Evaluated SSI Effects in a 3 DOF system on lateral and rocking springs. Considered frequency dependence of K functions.

**1969** **Nathan Newmark**  
Torsional effect in symmetric buildings due to the difference in the time of excitation at different points of the foundation. Called this the Tau Effect

**1970** **Sarrazin**  
Damping values are low for rocking and high for swaying. SSI effect reduces disp. Hysteretic damping is important.

**1976** **Scanlan**  
Well know large study on Tau Effect

**1977** **Veletsos**  
Interacting systems can be modeled via simple systems with modified periods and appropriate levels of damping